

## Contents of the solution data files

The textfiles `dataset1` to `dataset6` contain the results of the computations of 171 equilibrium solutions. Each solution has an identification number, which can be found below. Two lists belong to each solution: Their names are "daten" and "thetalist", respectively, followed by the identification number.

The first of these lists gives details of input and output, the second contains the computed values of the unknowns of the discretized problem. The latter list can be copied and inserted into the MAPLE code to start a new computation with similar parameters of the mixture and the film material.

### List thetalistxxxxxxx, meaning of the entries:

|                            |                                                                   |
|----------------------------|-------------------------------------------------------------------|
| Entry $1 \dots (r+1)(s+1)$ | values of dimensionless displacements $\theta$ of the grid points |
| Entry $(r+1)(s+1) + 1$     | value of $n$                                                      |
| Entry $(r+1)(s+1) + 2$     | value of $f$                                                      |
| Entry $(r+1)(s+1) + 3$     | value of $p_O$                                                    |
| Entry $(r+1)(s+1) + 4$     | value of $p_W$                                                    |
| Entry $(r+1)(s+1) + 5$     | value of $a$                                                      |

**List datenxxxxxxxx, meaning of the entries:**

|                  |                                                                                                     |
|------------------|-----------------------------------------------------------------------------------------------------|
| tnum             | identification number                                                                               |
| hh               | film thickness                                                                                      |
| n1i              | $(r + 1)$ , number of gridpoints in the $u$ -direction                                              |
| n2i              | $(s + 1)$ , number of gridpoints in the $v$ -direction                                              |
| L_o              | oil length of the mixture, $L_O$                                                                    |
| nu               | Poisson's ratio, $\nu$                                                                              |
| D0               | preferred curvature difference, $D_0$                                                               |
| H0               | preferred mean curvature, $H_0$                                                                     |
| a*theta[1]       | displacement of the grid point at $(u = 0, v = 0)$ in the direction of $\mathbf{m}$                 |
| ns*wc            | mean bending energy density, $\bar{w}$                                                              |
| a                | elementary length, $= e$ with tube3 and tube4, $= e/4$ with tube6, fixed with cylinders and spheres |
| po               | oil pressure, $p_O$                                                                                 |
| ac               | area of the elementary patch, $A_C$                                                                 |
| voc              | oil volume of the cell, $V_{OC}$                                                                    |
| vwc              | water volume of the cell, $V_{WC}$                                                                  |
| wc               | bending energy of the cell, $W_C$                                                                   |
| ns               | number $n$ of cells per unit area of the middle surface of the film                                 |
| f                | Lagrangean parameter $f$ , chemical potential                                                       |
| pw               | water pressure, $p_W$                                                                               |
| bed_o            | $L_O - l_O$ , measure of the oil excess                                                             |
| bed_w            | $L_W - l_W$ , measure of the water excess                                                           |
| ns*voc           | oil length of the structure, $l_O$                                                                  |
| ns*vwc           | water length of the structure, $l_W$                                                                |
| mean1,dev1       | mean value and standard deviation of $H$                                                            |
| mean2,dev2       | mean value and standard deviation of $D$                                                            |
| mean3,dev3       | mean value and standard deviation of $(w - \bar{w}) \text{tr } \mathbf{C}$                          |
| mean4,dev4       | mean value and standard deviation of $(p_O l_O + p_W l_W) \text{tr } \mathbf{C}$                    |
| mean5,dev5       | mean value and standard deviation of $\mathbf{M} : \mathbf{C}^2$                                    |
| mean6,dev6       | mean value and standard deviation of $\mathbf{T} : \mathbf{C}$                                      |
| mean7,dev7       | mean value and standard deviation of $p_n$                                                          |
| mean8,dev8       | mean value and standard deviation of $-\mathbf{T} : \mathbf{C} - p_n = \mathbf{q} \cdot \nabla_T$   |
| mean9,dev9       | mean value and standard deviation of $w$                                                            |
| a1c              | $\int_{A_C} 2H dA$                                                                                  |
| a2c              | $\int_{A_C} K dA$                                                                                   |
| evalf(-6/Pi*a2c) | defect, $\delta$                                                                                    |

## Identification numbers and properties of the solutions

Selection of solutions for the construction of figs. 5 to 11 ( $16 \times 16$  grid points, no volume constraint)

| <div>dataset1</div> : |       |       |          |          |          |          |
|-----------------------|-------|-------|----------|----------|----------|----------|
| $\nu = 0.9$           | $D_0$ | $H_0$ | $\omega$ | tube6    | tube4    | tube3    |
| (figs. 12)            | 1.0   | -0.1  | -0.0635  | —        | —        | 31690014 |
|                       | 1.0   | 0.0   | 0.0      | 61690001 | 41690001 | 31690012 |
|                       | 1.0   | 0.1   | 0.0635   | —        | —        | 31690010 |
|                       | 1.0   | 0.2   | 0.1257   | 61690004 | 41690003 | 31690009 |
|                       | 1.0   | 0.4   | 0.2422   | 61690008 | 41690005 | 31690007 |
|                       | 1.0   | 0.6   | 0.3440   | 61690012 | 41690007 | 31690005 |
|                       | 1.0   | 0.8   | 0.4296   | 61690016 | 41690010 | 31690003 |
|                       | 1.0   | 1.0   | 0.5      | 61690020 | 41690014 | 31690001 |
|                       | 0.8   | 1.0   | 0.5704   | 61690042 | 41690016 | 31690021 |
|                       | 0.6   | 1.0   | 0.6560   | 61690044 | 41690018 | 31690022 |
|                       | 0.4   | 1.0   | 0.7578   | 61690046 | 41690020 | 31690023 |
|                       | 0.2   | 1.0   | 0.8743   | 61690048 | 41690022 | 31690024 |
|                       | 0.1   | 1.0   | 0.9365   | —        | —        | 31690025 |
|                       | 0.0   | 1.0   | 1.0      | 61690050 | 41690024 | 31690026 |

**dataset2** :

| $\nu = 1.0$ | $e H_0$ | tube6    | $e H_0$ | tube4    | $e H_0$ | tube3    |
|-------------|---------|----------|---------|----------|---------|----------|
|             | 0.0     | 61612001 | 0.0     | 41612001 | 0.1     | 31612006 |
|             | 0.2     | 61612014 | 0.25    | 41612002 | 0.2     | 31612004 |
|             | 0.4     | 61612002 | 0.50    | 41612003 | 0.4     | 31612002 |
|             | 0.6     | 61612013 | 0.75    | 41612004 | 0.6     | 31612011 |
|             | 0.8     | 61612003 | 1.00    | 41612005 | 0.8     | 31612013 |
|             | 1.0     | 61612012 | 1.25    | 41612006 | 1.0     | 31612015 |
|             | 1.2     | 61612004 | 1.50    | 41612007 | 1.2     | 31612016 |
|             | 1.4     | 61612011 | 1.75    | 41612008 | 1.4     | 31612017 |
|             | 1.6     | 61612005 | 2.00    | 41612009 | 1.6     | 31612018 |
|             | 1.8     | 61612006 | 2.15    | 41612011 | 1.8     | 31612019 |
|             | 2.0     | 61612007 | 2.25    | 41612010 | 2.0     | 31612020 |
|             | 2.08    | 61612010 | 2.30    | 41612012 | 2.2     | 31612021 |
|             |         |          |         |          | 2.4     | 31612023 |
|             |         |          |         |          | 2.45    | 31612024 |

Note: The standard deviation of the mean curvature is given as an imaginary number in some of these lists. This results from a rounding error of a number under a square root, which should be zero. The complex number has simply to be replaced by zero. The problem is removed in the actual version of the code.

Selection of solutions for the construction of figs. 13 to 17 ( $16 \times 16$  grid points, oil volume constraint)

**dataset3** :

Case A of table 1 ( $\nu = 0.9$ ,  $D_0 = 0.0$ ,  $H_0 = 0.0$ )

| $L_O$ | tube6    | tube4    |
|-------|----------|----------|
| 0.30  | 61691089 | 41691089 |

Case B of table 1 ( $\nu = 0.9$ ,  $D_0 = 1.0$ ,  $H_0 = 0.0$ ,  $\omega = 0.0$ )

| $L_O$ | tube6    | $L_O$ | tube4    | $L_O$ | tube3    |
|-------|----------|-------|----------|-------|----------|
| 0.65  | 61691002 | 0.60  | 41691002 | 0.70  | 31691052 |
| 0.55  | 61691004 | 0.50  | 41691005 | 0.60  | 31691053 |
| 0.45  | 61691006 | 0.40  | 41691007 | 0.50  | 31691054 |
| 0.35  | 61691008 | 0.30  | 41691009 | 0.40  | 31691055 |
| 0.25  | 61691010 | 0.20  | 41691011 | 0.35  | 31691056 |
| 0.15  | 61691012 | 0.10  | 41691013 | 0.30  | 31691057 |
| 0.10  | 61691013 | 0.05  | 41691015 |       |          |
| 0.05  | 61691015 |       |          |       |          |

**dataset4** :

Case C of table 1 ( $\nu = 0.9$ ,  $D_0 = 1.0$ ,  $H_0 = 0.7$ ,  $\omega = 0.388$ )

| $L_O$ | tube6    | tube4    | tube3    |
|-------|----------|----------|----------|
| 0.30  | 61691061 | 41691061 | 31691061 |
| 0.25  | 61691062 | 41691062 | 31691062 |
| 0.20  | 61691063 | 41691063 | 31691063 |
| 0.17  | 61691064 | 41691064 | 31691064 |
| 0.14  | 61691065 | 41691065 | 31691065 |
| 0.11  | 61691066 | 41691066 | 31691066 |
| 0.08  | 61691067 | 41691067 |          |
| 0.05  | 61691068 | 41691068 |          |

**dataset5** :

Case D of table 1 ( $\nu = 0.9$ ,  $D_0 = 0.0$ ,  $H_0 = 0.7$ ,  $\omega = 1.0$ )

| $L_O$ | tube6    | tube4    | tube3    |
|-------|----------|----------|----------|
| 0.45  | 61691038 |          |          |
| 0.44  |          |          | 31691024 |
| 0.42  |          |          | 31691026 |
| 0.40  | 61691039 | 41691039 |          |
| 0.38  |          |          | 31691027 |
| 0.35  | 61691040 | 41691040 | 31691028 |
| 0.30  | 61691041 | 41691041 | 31691029 |
| 0.25  | 61691042 | 41691042 | 31691031 |
| 0.20  | 61691043 | 41691043 | 31691033 |
| 0.17  | 61691044 | 41691044 | 31691034 |
| 0.14  | 61691045 | 41691045 | 31691035 |
| 0.12  |          |          | 31691036 |
| 0.11  | 61691046 | 41691046 |          |
| 0.08  | 61691047 | 41691047 |          |
| 0.05  | 61691048 | 41691048 |          |

Case E of table 1 ( $\nu = 1.0$ ,  $H_0 = 0.0$ )

| $L_O$ | tube6    | tube4    |
|-------|----------|----------|
| 0.30  | 61611009 | 41611009 |

Case F of table 1 ( $\nu = 1.0$ ,  $H_0 > 0.0$ )

The data are identical with those of the case without volume constraint,  $\nu = 1.0$ .

**dataset6** :

Selection of solutions for the construction of figs. 18 ( $16 \times 16$  grid points,  $\nu = 0.9$ ,  $D_0 = 1.0$ ,  $H_0 = 0.0$ ,  $\omega = 0.0$ , oil and water volume constraint,  $L_W/L_O = 1.15$ )

| $L_O$ | tube4    |
|-------|----------|
| 0.44  | 41695006 |
| 0.36  | 41695008 |
| 0.28  | 41695011 |

Solution for the construction of fig. 19a: 61690050, of fig. 19b: 61690021

Solutions for the construction of fig. 20 ( $D_0 = 0$ ,  $H_0 = 1$ , no volume constraint):

| $\nu$  | tube6    | tube4    | tube3    |
|--------|----------|----------|----------|
| 0.90   | 61690050 | 41690024 | 31690026 |
| 0.83   |          |          | 31680056 |
| 0.80   | 61680050 | 41680024 |          |
| 0.65   | 61660003 | 41660024 | 31660056 |
| 0.50   | 61650003 | 41650024 | 31650056 |
| 0.3333 | 61632011 | 41630024 | 31630056 |

Solution for the construction of fig. 21: 61691056.

Tests with rectangular meshes:

| mesh           |          | mesh          |          | error of the defect    |
|----------------|----------|---------------|----------|------------------------|
| $16 \times 16$ | 61690050 | $4 \times 71$ | 69990052 | no improvement         |
| $16 \times 16$ | 31612006 | $41 \times 7$ | 39912026 | remarkable improvement |